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Russian Billiards: Table Difficulty

Part 1

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Who was the first «to voice» the concerns about the billiard table difficulty? Apparently, it was a Russian Billiards theorist Anatoly I. Leman who wrote the book «Theory of billiard game». Its first edition was published in 1885, and by this time many books on the topic of billiards have been published abroad. Nevertheless, the question about pockets and table difficulty was not raised (in any case, in those primary sources, which I was able to read). And it's not surprising, because the pocket opening (pocket mouth size) on tables for Pool, English Billiards and Snooker significantly exceed the diameter of the balls used (in Russian Billiards the same parameters are not simply comparable with each other, but they are almost identical). Only quite recently, foreign billiard theorists have started to attach importance to the research in pockets and table difficulties. Thus, in November 2013 David Alciatore (also known as Doctor Dave) published some results of table difficulty calculations posted on the website *Billiards and Pool Principles, Techniques, Resources* in the article *Billiard University (BU) – Part IV: Table Difficulty*.

In his book A.I.Leman noted that difficult tables appeared in Russia not at once. Initially popular were billiard games in which, in addition to potting the balls into the pockets, a cannon – caroming the cue ball off one object ball into another – was considered a successful attack action. And only after most of the players started to prefer tables on which the ball fell into pocket «only under very sure shot», tables, the pocket mouth size of which barely exceeded the balls' diameter, became widespread. The majority of billiard amateurs still believe that the difficulty of the table is determined only by the ratio of the diameter of the ball to a pocket opening. However, even Leman no longer shared this view. He said: « At first glance it is easy to make difficult every billiard table: we need only take the balls that would have hardly entered into the pockets. It is true, but it would be a relative difficulty. True difficulty is in observation of a mathematical proportionality between the table size, pocket opening and the size of balls' diameter …». And immediately afterwards, Leman explains his idea – the reason for the said proportionality: «The following important law for all Russian billiard games should be kept in mind**: it is required that every ball falls with a fast shot in any pocket, and it would be impossible to pot a ball into side pocket when it is rolling along cushion**». (Note: hereinafter the text is highlighted as in the original). I am not here to criticize the last of the above Leman's statements, but I'll go on considering the position, formulated by him, consecutively examining components of the true, in his opinion, difficulty, such as size of the table, pocket mouth and balls used for playing. As to the size of the table Leman said: «A good billiard table should be especially **great**. This is its main advantage and it is so important that no matter how precise, elegant and difficult a small table is, a great player will not play on it. And this is true. On a small table other shots and other calculations are used. (*Note: Leman believed that the length of the playing field on large billiard table is 5 arshins (1 arsshin = 0.711 m), and lies in the range of 4 to 4.5 arshins for small tables. If a recount, it becomes clear that by large billiards Leman understood tables, similar in size to the current 12-foot table, and as to small tables – those, close to the modern 10-ft.*). A player, who excellently plays on a big billiard table, plays on a small table as confidently and freely , but he often leaves easy shot for opponent and being irritated spots the weakest player is obviously winning the frame (game). And it's easy to see that if for large tables it is a good winning back, then for small tables it becomes a hanger (or «duck»). In short, the basic law of billiard game is the most obvious here: **the more forged, lighter and smaller the table, the more balanced the forces of a good and a bad player are.** That is why top-notch player will never play on a bad billiard table». Leman offered to choose diameter balls for Russian

Billiards between 67 to 72 millimeters. I quote: «The magnitude of the balls has a decisive character for grace or rudeness of billiard game. It should be remembered that too small balls, less than 67 mm in diameter, are unseemly for a good player. Playing with these balls is too insignificant. This game is not exiting, not a pleasure. It is hard to play with too large balls (more than 72 mm) as well. It will be a bowling, and not a billiard. The larger the diameter of a ball, the harder it is to perform all the shots. In French billiards with small balls and light cues, all strokes are extremely easy. The fewer types of shots, the rougher game is». Dimensions of balls and pocket openings used in Russian Billiards quite closely linked. Leman asserted that «... **the diameter of the ball should be** slightly smaller than the pocket mouth size. This is a necessary and a substantial amendment to the selection of balls to pockets». In addition he said, «But it is also important for the **pocket mouth size** to be consistent with the **billiard table size**». The foregoing should not be interpreted in the way that Leman advocated «restraint» or, in other words, excessive difficulty of Russian Billiards. On the contrary, he knew that the reverse side of excessive difficulty is the limited technical and tactical techniques that can be used in the game. This is confirmed by the following words from his book: «Billiard tables …are often so difficult that good players play a game for about an hour. It means that on these tables all various elements of a game are destroyed and everything is only sacrificed to a precision aiming. This game should be called rude. Such billiard game, as narrow as it is, approaches to bowling». On the other hand, Leman did not welcome all sorts of improvements aimed at undue reduction of table difficulty. For example, he wrote: «Feeling the shortcomings of their tables, manufacturers began to make the corner pocket mouths too wide, a side pocket very tight; and cushions much firmer, which allowed you to develop the force of shots. But it did not help». However, Leman's position on one of the improvements seems a little bit strange: «Freyberg began somewhat to increase the width of the table relative to its length. Then the pockets were indeed made **more open** and game became easier». (*Arnold Freiberg – a very famous Russian billiard manufacturer*). Strangeness, in my opinion, is that if the author welcomes and commends these changes, then he would have to remove (or paraphrase) the following lines in the beginning of the chapter: «The billiard table plane consists of **two squares** stacked together. This ancient form of the table is based on a thorough study of the billiard game». Yet summing up, it can be argued that Leman advocated a reasonable and sufficiently rigid coherency of size of pockets, balls and playing surface of a billiard table. In order to achieve «correct behavior» of pockets under such very strict restrictions, he offered to find a way to competent cutting of pocket configuration: «On the accurate billiard table exact purity and beauty of the game depends on a reasonable cut of pockets ...». Following the same principle formulated by him «pockets should be cut **strictly in proportion** to table size», Leman proposed a specific way for cutting corner pocket with reduced shelf depth. Basing on these words and on a few above-mentioned quotations you can conclude that Leman does not distinguish between the concepts of «pocket difficulty» and «pocket intake capacity» («infectivity») and put them into a coherent whole.

Recently, on online forum (site Billiard-Online) I perused the topic «*On the cushions and pockets rubber*» where the messages were dated from the end of January to early April 2010. It turned out that in the main message («On the Table for Russian Billiards») a number of considerations by Vitaly Arkhipov (aka Habib) were compactly represented, and some of them were close to me in spirit, in understanding aspects of billiard game. I noted a certain systematic approach that could not fail to please me – in due time I was taught to think (or not taught, it is another question) exactly in this way. In particular, I highlighted the consideration of the possibility and usefulness of obtaining numerical estimates of the local difficulty for specific pocket on the table with known dimensions for a particular ball, located in the playing surface at the given coordinates. Furthermore, it seemed a quite logical and consequent proposal to assess difficulty of entire table by averaging the sum of local estimates calculated for all within possible points of location of the ball. However, unlike me, not all forum visitors reacted positively to the ideas of V.Arkhipov. As it often happens, in addition to various arguments an argument «you're a fool yourself» passed in the discussion as a red thread. Disputants urged each other to carry out numerical calculations and threatened to report the results to the general court. But, unfortunately, it ended just chatter. So I decided to do everything myself.

In addition to praising V.Arkhipov I'll present some negative position:

1. Quote: «The Leman's idea of mathematical proportionality of table, pockets and balls sizes can be explained as follows». It seems that some ideas should not be ascribed to Leman, and you should not explain in your own way what he did not say. Leman clearly explained what he meant by this proportionality (see above), and its interpretation is far from local and integral estimates of table difficulty.

2. Quote: «True table difficulty is an integral (consolidated) geometric parameter uniquely determined by the size of the balls, pockets and sides of the field of billiard table. With this parameter, you can objectively compare the tables of different sizes together, and compare tables for Pool, Snooker and Russian Billiards». (*Note: The selection underlined - my*).

As to uniqueness: I understand it as a strict definition of the concept, which means that it has a single precise definition. In fact, there is no uniqueness in this case; a somewhat different definition of difficulty will be presented below, which has the right to exist, as well as many other possible definitions.

Speaking about the objectivity of the comparison applied to the subject, it can be assumed that the resulting evaluation of the object characteristics does not depend on the subject. But this is not so – not least because there is no uniqueness. It seems that V.V.Generalov had this in mind, when he spoke about the billiard difficulty: «Pocket difficulty, in general, is of a category ... – a girl is beautiful or not». (*Vladimir V. Generalov – a very famous Russian Billiards theorist, died prematurely in 2010*).

Of course, it is possible to compare Pool, Snooker and Russian Billiards tables by difficulty, but is it worth to do it? Firstly, what received evaluation can show? Only that a Pool table, say, twice is less difficult than a table for Russian Billiards and Snooker table – is less than one and half times? Well, what of it? In fact, we have already had some idea about it, though based on the senses, that is on expert and not on numerical estimates. And secondly, Pool and Snooker tables have pockets whose geometry differs significantly from the geometry of pockets for Russian Billiards, with rather «sharp» knuckles (points). Therefore, it makes sense to compare the likely tables' intake capacity rather than their difficulty.

Speaking about the billiard table difficulty, you need to have a clear idea about the concept of intake capacity. Here is a long quotation from V.Arkhipov's title post: «What is the intake capacity in Russian Billiards? Ball's potting into the pocket at the exact shot is determined not only by the ratio of the ball diameter to the size of pocket opening. Even if the pocket opening is less than the diameter of the ball, the ball can fall into a pocket, being reflected from the pocket knuckle, pre-pushing through it. At the same time whether the ball drops into a pocket or not will depend on the rubber properties and geometry of pocket point, surface condition of the ball and cushion cloth, translational and angular velocity of the ball. Mathematically pocket intake capacity parameter P can be described by the ratio of table's zone area, from which it is possible potting a ball into the pocket, to the total area of the playing surface. The entire table intake capacity is calculated the arithmetic mean of all six pockets intake capacities. As well as true table difficulty, the table intake capacity is an integral (consolidated) parameter, not purely geometric, but rather physical and geometric. This can be estimated only on the basis of experimental and statistical data collected on a particular table. This parameter can also be used for objective comparison tables of different sizes and manufacturers together, and compare tables for Pool, Snooker and Russian Billiards as well». We noted above that Leman did not separate difficulty from the intake capacity. V.Arkhipov also clearly distinguishes face section. Here's how he offers to find table difficulty estimation: «To a certain position of the ball having a diameter D, located on the pocket centerline at a distance L from the center of the pocket with opening of H, we can introduce difficulty parameter S, equating it to the angular error of shot, i.e. $Arcsin\left(\frac{H-D}{L}\right)$ (sufficiently accurate approximation). Dimensionless quantity was to need to bring it to the visible angle of pocket opening, i.e. $S = Arcsin\left(\frac{H-D}{L}\right)/Arcsin\left(\frac{H}{L}\right)$. If you break a table into small squares, calculate the difficulty values S for them (including reducing the visible pocket opening H in case the ball is not located on the centerline of the pocket) and calculate the arithmetic mean, we obtain the numerical pocket difficulty parameter S on a particular table. Summing values for six pockets and dividing by 6, we obtain the parameter of true table difficulty». It is easy to see that when the diameter of the ball is greater than the apparent width of pocket opening, the value of difficulty will be negative. By the way, this fact simply infuriated L.Baltsev – well, he does not perceive negative values! As for me it did not seem anything out of the ordinary, but later I «went» from negative – this way it is more convenient. Much more important is a negative value S in V.Arkhipov's formula that in some cases at $(H-D) < 0$, the ball can still get into the pocket, but in this case the difficulty parameter indicates the opposite. In general, the above formula immediately struck me as weird, and it (in addition to the above) also contributed to my independent calculations. Unlike V.Arkhipov I thought it appropriate to identify the pocket difficulty taking into account the given projection of the ball on pocket opening at the moment of contact with the far pocket point, which is the minimum necessary for a possible potting the ball into the pocket (which is possible, but not compulsory!). In this sense, this approach partly combines elements of the concepts of pocket's «difficulty» and «intake capacity».

Approach 1

Here and below we assume that if the ball is in contact with near pocket point it can get into pocket if the contact occurred in passing. Contact is possible with the far point and as a «tight» contact; but it must be done under an essential condition: the width of projection of the ball, attributable to the side of the pocket opening, should not be less than the radius of the ball.

Look at Figure 1. Here we use the following designations: B – near (to the initial position of the ball) pocket point; D – far pocket point; O – the initial location of the center of the ball; R – ball radius; $BD = H$ – width of pocket opening (pocket mouth size). We denote $\rho_B = OB$ distance from the initial position of the ball to the near point and through $\rho_D = OD$ – the distance from the initial position of the ball to far point. Let X_O, X_B, X_D, Y_O, Y_B, Y_D – X and Y coordinates of points O, B and D. Starting rectangular coordinate system can be positioned at any point on the table surface; it seems that the very «convenient» location is the intersection point of the long and short sides in the opening of one of the corner pockets.

Having information about the locations of the ball and the pocket, find the values of ρ_B and ρ_D:

$$\rho_B = \sqrt{(X_O - X_B)^2 + (Y_O - Y_B)^2}\,, \qquad (1)$$

$$\rho_D = \sqrt{(X_O - X_D)^2 + (Y_O - Y_D)^2}\,. \qquad (2)$$

Define the angle γ_Σ between the directions from point O to pocket points. To do this, we use law of cosines for triangle OBD

$$H^2 = {\rho_B}^2 + {\rho_D}^2 - 2\,\rho_B\,\rho_D\,Cos\,\gamma_\Sigma\,,$$

hence

$$\gamma_\Sigma = Arccos\left(\frac{{\rho_B}^2 + {\rho_D}^2 - H^2}{2\,\rho_B\,\rho_D}\right). \qquad (3)$$

Figure 1 shows that

$$\gamma_\Sigma = \gamma + \gamma_B + \gamma_D \ , \tag{4}$$

$$\gamma_B = Arctg\left(\frac{R}{\rho_B}\right). \tag{5}$$

We denote by ξ ratio of the width of the projection of the ball falling on pocket opening, to the diameter of the ball. In other words, ξ is the fraction of the diameter of the ball, which when in contact with the far point is on the pocket opening. It is clear that to potting the ball into the pocket should be performed restriction $0.5 \leq \xi \leq 1$. Upon half-ball impact with the far point observed condition $\xi = 0.5$, and the case $\xi = 1$ corresponds to the contact of the ball with far point in passing.

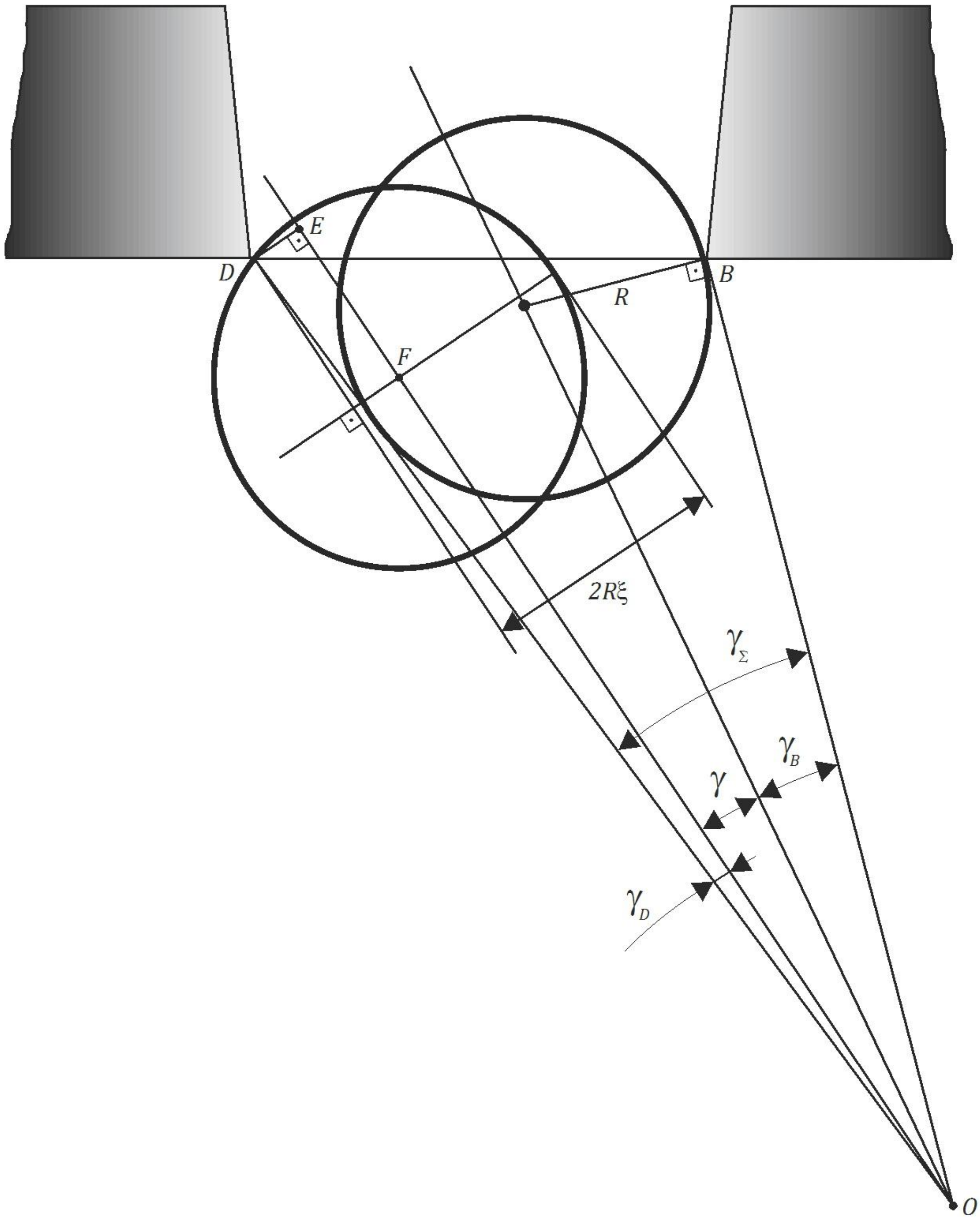

Fig.1. Potting the ball into the pocket after casual contact with the near point and after contact with the far point.

Find an angle γ_D:

$$Sin\ \gamma_D = \frac{DE}{\rho_D} = \frac{2R\xi - R}{\rho_D}. \quad (6)$$

Finally, we define the angle γ from the relation (4):

$$\gamma = \gamma_\Sigma - \gamma_B - \gamma_D\ , \quad (7)$$

where γ_Σ and γ_B can be found using (3) and (5), and γ_D can be calculated using relation (6). Angle γ indirectly characterizes difficulty of considered pocket for a particular initial position of the ball. It determines the area of the playing surface, which must be located within the trajectory of the ball, so that he can get into the pocket. The greater the angle γ, the wider the zone indicated. Angle γ can be interpreted as the parameter that determines margin of the angular error in the movement of the ball, allowed to ball still had a chance to get inside pocket.

We introduce the normalized index of pocket difficulty, which can be called its local difficulty:

$$S_\gamma = 1 - \frac{\gamma}{\gamma_\Sigma}. \quad (8)$$

When $\gamma = 0$ local pocket difficulty takes the value $S_\gamma = 1$ (or 100% if expressed as a percentage). This means that this is the necessary condition for the specified fraction of the ball diameter attributable to pocket opening in contact with far point, but in this case the local difficulty will be the maximum – the ball will go through the pocket opening with casual touching of near point. Ratio γ/γ_Σ in the right part of (8) shows – what the relative «freedom» in the passing through the pocket opening there compared with the maximum difficulty $S_\gamma = 1$. In fact, the relation (8) provides the ability to calculate the pocket difficulty as V.Arkhipov suggested – by the ratio of the angular size of the zone of possible positions of the ball when passing to the visible angular size of the pocket opening.

To calculate the pocket difficulty in relation to the entire playing surface (in other words – the integral value of difficulty), you must «sort out» all possible points of the playing field, compute for each of them local indicators of difficulty (according to (8)), summarize the results and average them. It seems that here (and in the approaches to be presented below) we should exclude from the calculation those points of the initial position of the ball, which under no condition be able to get directly to the considered pocket. These are provisions of the ball close to the cushion on which the side pocket is located. In this case the impossibility of getting the ball inside the pocket is not due to pocket difficulty – because no matter how wide it can be the ball will not be able to penetrate in it.

While calculating the values of S_γ, it is necessary to take into account two possible special cases,

a. After passing the pocket opening with casual touch of near point, ball diameter share attributable in contact with the far point, less than a predetermined value $2R\xi$. *Respectively, the ball can't get inside the pocket.*

The condition under which the ball can't penetrate into the pocket, according to Figure 2, can be represented as $W < 2R\xi$. Determine the value of W. To do this, find an angle DBO from the triangle DBO (using the law of cosines):

$$Cos < DBO = \frac{{\rho_B}^2 + H^2 - {\rho_D}^2}{2\ \rho_B\ H}.$$

Determine an angle MBO from the triangle TBO:

$$Cos < MBO = \frac{R}{\rho_B} .$$

Since $W = MB = H\ Cos < DBM$ and $< DBM = < DBO - < MBO$, then

$$W = H\ Cos\left[Arccos\ \frac{{\rho_B}^2 + H^2 - {\rho_D}^2}{2\ \rho_B\ H} - Arccos\ \frac{R}{\rho_B}\right] . \qquad (9)$$

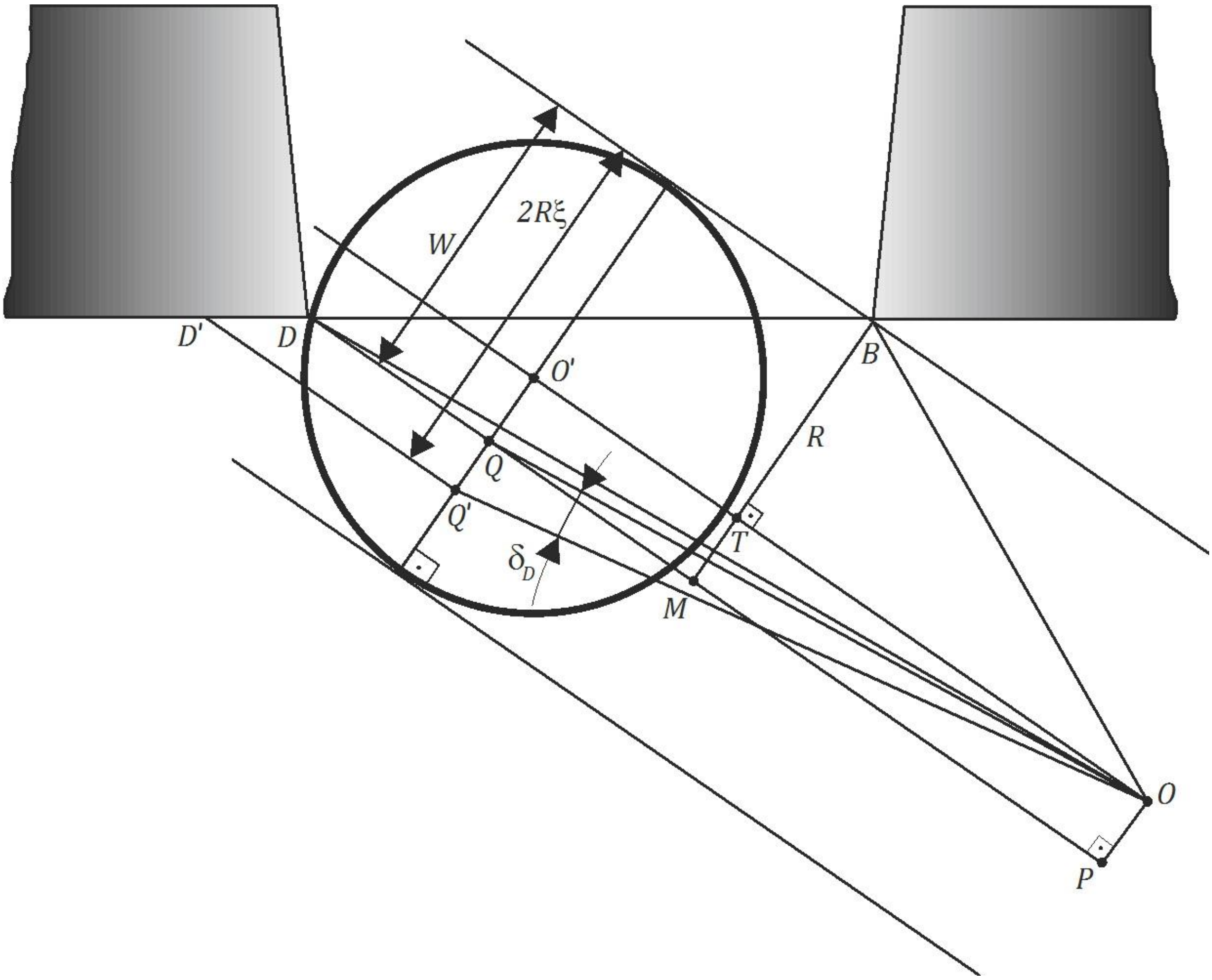

Fig.2. Hit the ball into the far point after contact with near pocket point in passing.
Case $R < W < 2R\xi$.

Find an equation, with which you can calculate the angle δ_D, characterizing the deviation of W from setpoint $2R\xi$. From the triangle MDB define MD, and from $QDO' - DQ$:

$$MD = \sqrt{H^2 - W^2} , \qquad DQ = \sqrt{R^{\ 2} - (W - R)^2} .$$

Since $MQ = MD - DQ$, then

$$MQ = \sqrt{H^2 - W^2} - \sqrt{R^2 - (W - R)^2} .$$

Of the triangle OTB can be seen that

$$OT = \sqrt{{\rho_B}^2 - R^2}\ ,$$

so it is possible to express the distance OO' from the initial position O to the position of the center of the ball O' at the moment of contact with the far point:

$$OO' = OT + TO' = OT + MQ = \sqrt{{\rho_B}^2 - R^2} + \sqrt{H^2 - W^2} - \sqrt{R^2 - (W-R)^2}\ .$$

Using triangles $OO'Q'$ and $OO'Q$ we finally obtain ($\delta_D = < Q'OO' - < QOO'$):

$$\delta_D = Arctg\left(\frac{2R\xi - R}{OO'}\right) - Arctg\left(\frac{W-R}{OO'}\right).$$

Pocket difficulty for this case is not calculated by the relation (8), but as follows:

$$S_\gamma = 1 + \frac{\delta_D}{\gamma_\Sigma}\ . \tag{10}$$

In (10), in contrast to (8), there is a sign «+» instead of «–». This is a consequence of the fact that in relation to the far point D the angle δ_D located opposite shift W. In view of this, in such cases, the pocket difficulty takes values larger than unity.

The case of ball collision with the far point when $R < W < 2R\xi$ depicted in Figure 3. If you do the computations as given above, it is easy to make sure that all the relations retain their form (while the value of $Arctg\left(\frac{W-R}{OO'}\right)$ we'll need to determine within $(-\frac{\pi}{2}, 0)$). Nothing in the calculations will not change when considering the collision of the ball to the far point of corner pocket, rather than the side pocket.

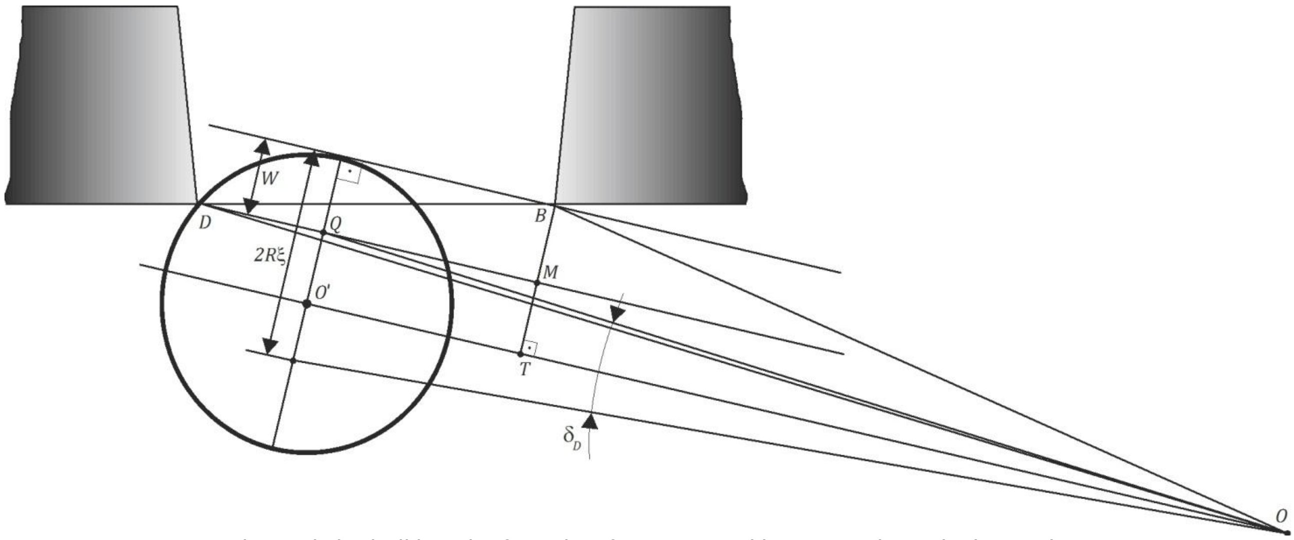

Fig.3. Hit the ball into the far point after contact with near pocket point in passing. Case $W < R < 2R\xi$.

b. <u>Initial location of the ball coincides with the pocket centerline</u>.

In such cases, the concepts of «near» and «far» point coincide. This means that the contact with both (the one and the other knuckle) is acceptable. In view of this, the angle should not be defined by the relation (7), but as follows:

$$\gamma = \gamma_\Sigma - 2\,\gamma_D = \gamma_\Sigma - 2\,\gamma_B\ . \tag{11}$$

Pocket difficulty with respect to the entire playing surface is an integral characteristic. Its evaluation can be found by making calculations for each pocket for $n \cdot m$ points in the table's plane with coordinates (X_{O_i}, Y_{O_i}), $i=\overline{1,n}$, $j=\overline{1,m}$:

$$S_k = \frac{1}{nm} \sum_{i=1}^{n} \sum_{j=1}^{m} S_{ij}\ , \tag{12}$$

where k – pocket number, $k=\overline{1,6}$; $S_{ij} = S_\gamma(X_{O_i}, Y_{O_i})$ – pocket difficulties with the respect to the initial ball position with coordinates (X_{O_i}, Y_{O_i}). It is understood that the estimation is more accurate the greater values are integers n and m. However, it should be understood that increasing the number of points $n \cdot m$ will lead to an increase in computational cost.

Knowing values S_k, $k=\overline{1,6}$, entire table difficulty can be defined as follows:

$$S = \frac{1}{\sum_{k=1}^{6} \alpha_k} \sum_{k=1}^{6} \alpha_k S_k\ , \tag{13}$$

where α_k, $k=\overline{1,6}$, $0 \le \alpha_k \le 1$ – weighting factors that determine the «importance» of each of the pockets to the variety of billiard game. In the particular case – being equally pockets:

$$S = \frac{1}{6} \sum_{k=1}^{6} S_k\ , \tag{14}$$

The results of calculations performed in the first approach, presented in Figure 4. As starting data used were the following quantities: $R = 34$ mm; $H = 72$ mm – for corner pockets; $H = 82$ mm – for side pockets; dimensions of the playing field 12-foot table – $3550{\times}1775$ mm; dimensions of the playing field 10-foot table; cushion nose height – 42 mm; $n = 2000$; $m = 1000$.

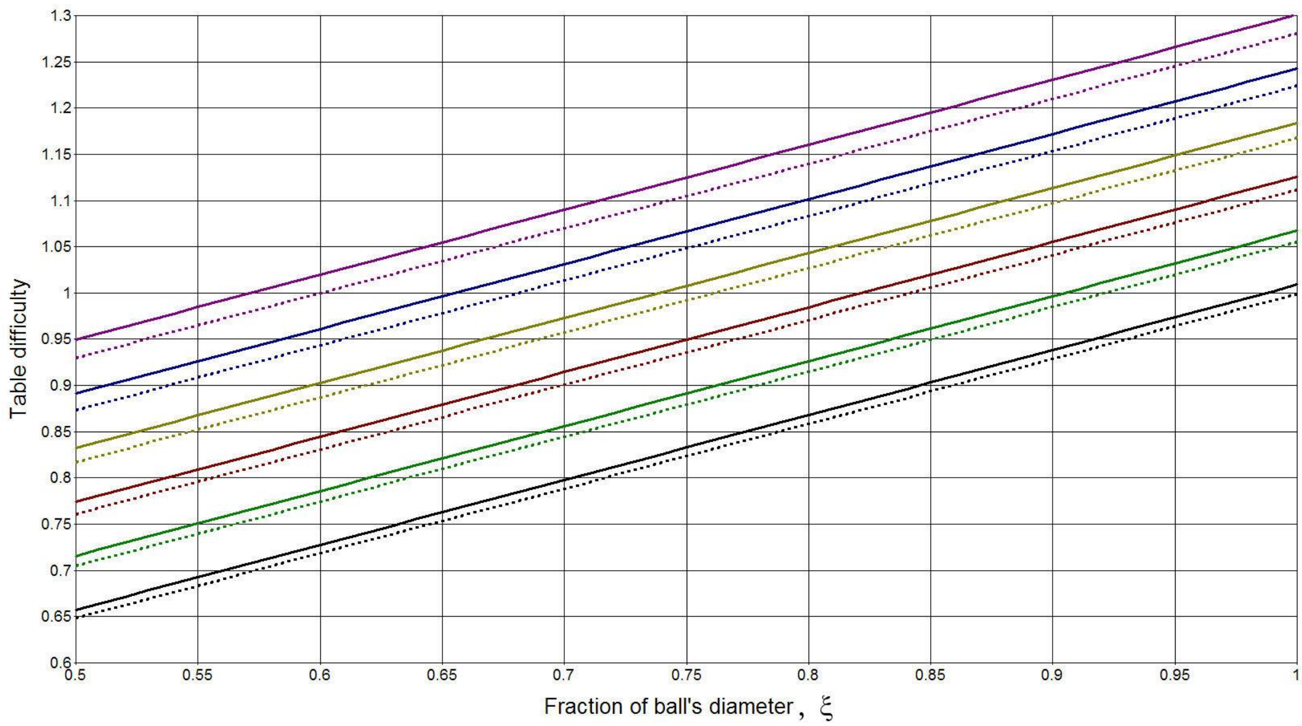

Fig.4. Difficulties for 12-foot and 10-foot billiard tables, calculated using approach 1.

The solid lines show the difficulty of 12-foot table, and dashed – 10-feet. Black lines (bottom in Figure 4) correspond to the value of $\xi = 0.5$ for the side pocket, purple – $\xi = 1$, and the lines of other colors represent intermediate (from 0.6 to 0.9) for the mean values of ξ for side pocket. It can be seen that depending $S(\xi)$ are essentially linear, so will not be difficult, if necessary, numerically processed (e.g., by least squares method) to obtain calculation results and empirical formulas for calculating the difficulty of 12-foot and 10-foot tables. I note that the difficulty of the 10-foot table was only a couple of percent less than difficulty of 12-foot table, although initially it seemed to me that the difference would be much more sensitive.

Approach 2

Local difficulty for pockets can be assessed in other ways. We consider an approach in which, instead of angles used to assess the above linear values are applied . Let us first consider the case when the ball without touching the near point, touches the far knuckle when $R < 2R\xi$ (Figure 5). If the pocket was more narrow and near point was located at B', then touching it would happen in passing. This means that the distance DB' can be regarded as a virtual pocket width, in this case, allowing the ball potting. Locate the value of DB', which we use in the future to assess the local pocket difficulty.

Angle EDO was previously defined (see (6)): $Sin < EDO = (2R\xi - R)/\rho_D$. Applying the law of cosines to the triangle ODB, we find angle ODB:

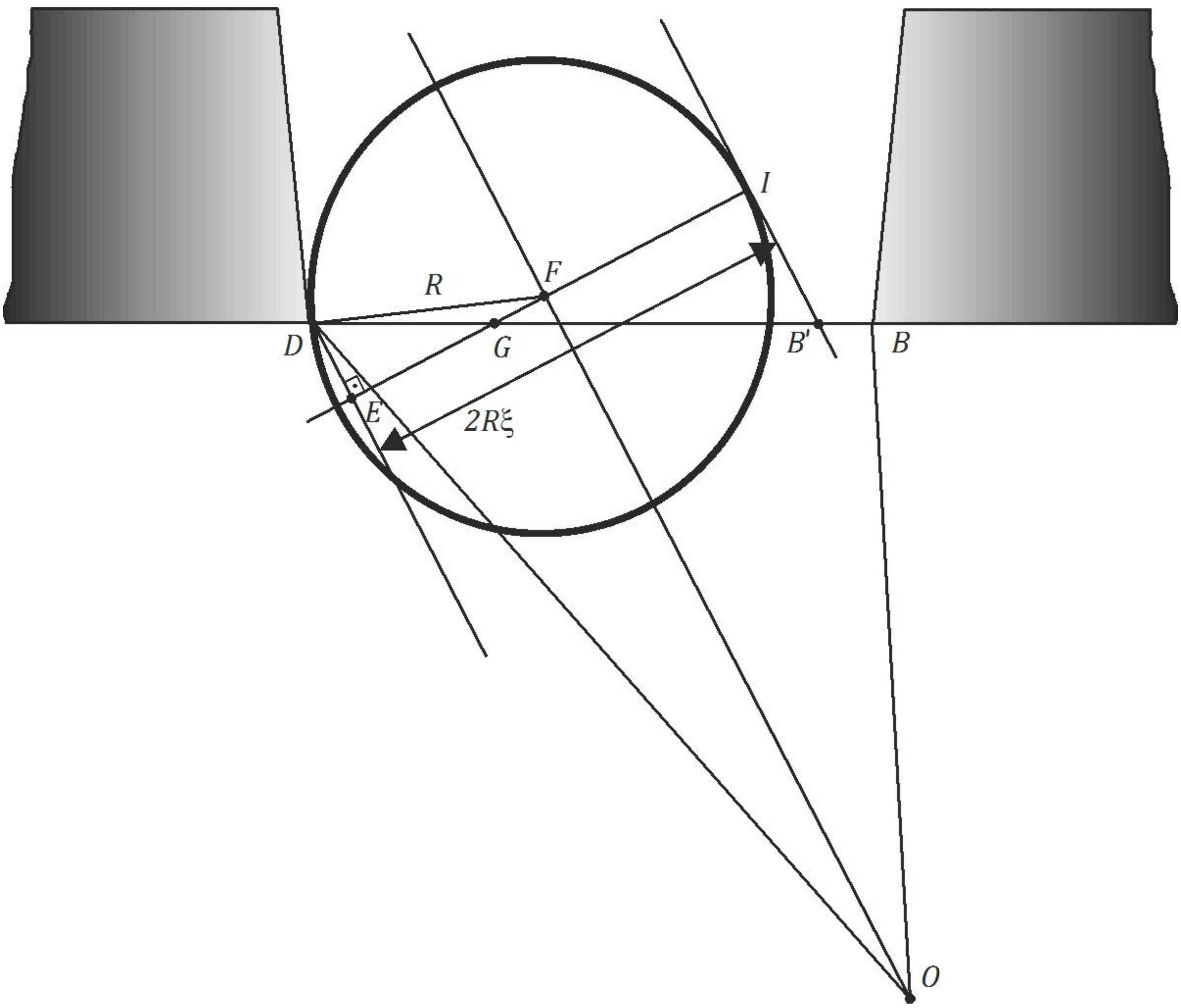

Fig.5. Hit the ball into the far point without contact with near pocket.

$Cos < ODB = \frac{\rho_D{}^2 + H^2 - \rho_B{}^2}{2\ \rho_D\ H}$. You can now express the angle EDG:

$$< EDG = < EDO + < ODB = Arcsin\ \frac{2R\xi\ - R}{\rho_D} + Arccos\ \frac{\rho_D{}^2 + H^2 - \rho_B{}^2}{2\ \rho_D\ H}.$$

From the similarity of triangles EDG and $IB'G$ express proportion $\frac{DG}{GB'} = \frac{EG}{GI}$, which with the help of relation $GI = 2R\xi\ -EG$ to the form $\frac{DG}{GB'} = \frac{EG}{2R\xi\ -EG}$. Hence we find

$$GB' = DG\ (2R\xi\ -EG)\ /\ EG.$$

Express DB':

$DB' = DG + GB' = DG + DG\ (2R\xi\ -EG)/EG = DG\ (1 + \frac{2R\xi\ -EG}{EG}) = DG\ (1 + \frac{2R\xi}{EG} - 1) = DG\ \frac{2R\xi}{EG}$. Because of the triangle EDG can be concluded that $\frac{EG}{DG} = Sin < EDG$, then $\frac{DG}{EG} = \frac{1}{\frac{EG}{DG}} = \frac{1}{Sin < EDG}$. Hence $DB' = \frac{2R\xi}{Sin < EDG} = \frac{2R\xi}{Sin\ (Arcsin\ \frac{2R\xi - R}{\rho_D} + Arccos\ \frac{\rho_D{}^2 + H^2 - \rho_B{}^2}{2\ \rho_D H})}$.

Knowing the value of DB', we can estimate the local pocket difficulty: $S_\gamma = 1 - \frac{B'B}{DB} = 1 - \frac{DB\ -\ DB'}{DB} = 1 - 1 + \frac{DB'}{DB} = \frac{DB'}{DB} = \frac{DB'}{H}$. Or finally:

$$S_\gamma = \frac{2R\xi}{H\ Sin\ (Arcsin\ \frac{2R\xi - R}{\rho_D} + Arccos\ \frac{\rho_D{}^2 + H^2 - \rho_B{}^2}{2\ \rho_D H})}. \quad (15)$$

Naturally, such an approach is necessary to consider the special cases mentioned above. When $W < 2R\xi$ (see Figure 2) as a virtual width of the pocket, allowing in this case drop the ball, you can consider the distance BD'. In other words, if pocket opening located between points B and D', then the ball could get into it, as a necessary condition would be satisfied $W = 2R\xi$. Easy to see that $BD' = 2R\xi/(Cos < DBM)$, and the angle of DBM has been found above (see (9)):

$$Cos < DBM = Cos\left[Arccos\ \frac{\rho_B{}^2 + H^2 - \rho_D{}^2}{2\ \rho_B\ H} - Arccos\ \frac{R}{\rho_B}\right].$$

Evaluation of local pocket difficulty in this case is defined as follows:

$$S_\gamma = 1 + \frac{DD'}{H} = 1 + \frac{BD'\ -\ H}{H} = 1 + \frac{BD'}{H} - 1 = \frac{BD'}{H}.$$

In cases where the initial position of the ball is on the pocket centerline (see Figure 6), the formula for local difficulty takes the form:

$$S_\gamma = 1 - \frac{2BB'}{DB} = 1 - \frac{2(DB - DB')}{DB} = 1 - 2\ (1 - \frac{DB'}{DB}) = 1 - 2 + 2\ \frac{DB'}{H} = 2\ \frac{DB'}{H} - 1\ ,$$

and DB', as above, from the relation

$$DB' = \frac{2R\xi}{Sin < EDB} = \frac{2R\xi}{Sin\ (Arcsin\ \frac{2R\xi - R}{\rho_D} + Arccos\ \frac{\rho_D{}^2 + H^2 - \rho_B{}^2}{2\ \rho_D H})}.$$

The results of the calculations performed according to the second approach, depicted in Figure 7. Calculations used the same raw data as before. Obtained by the second approach table difficulty values are slightly higher than the corresponding values arising from approach 1.

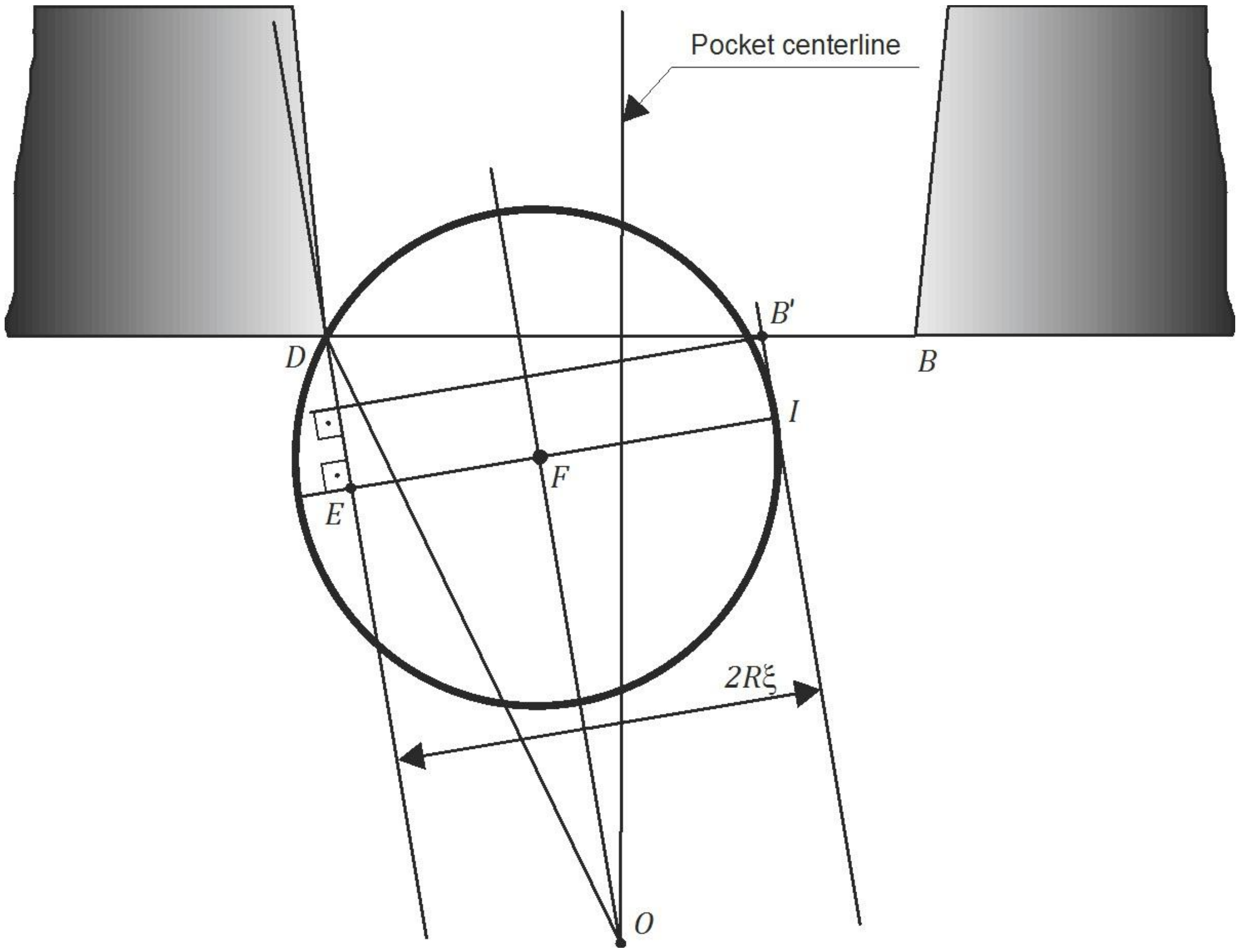

Рис.6. Hit the ball into the point after moving from its initial position on pocket centerline.

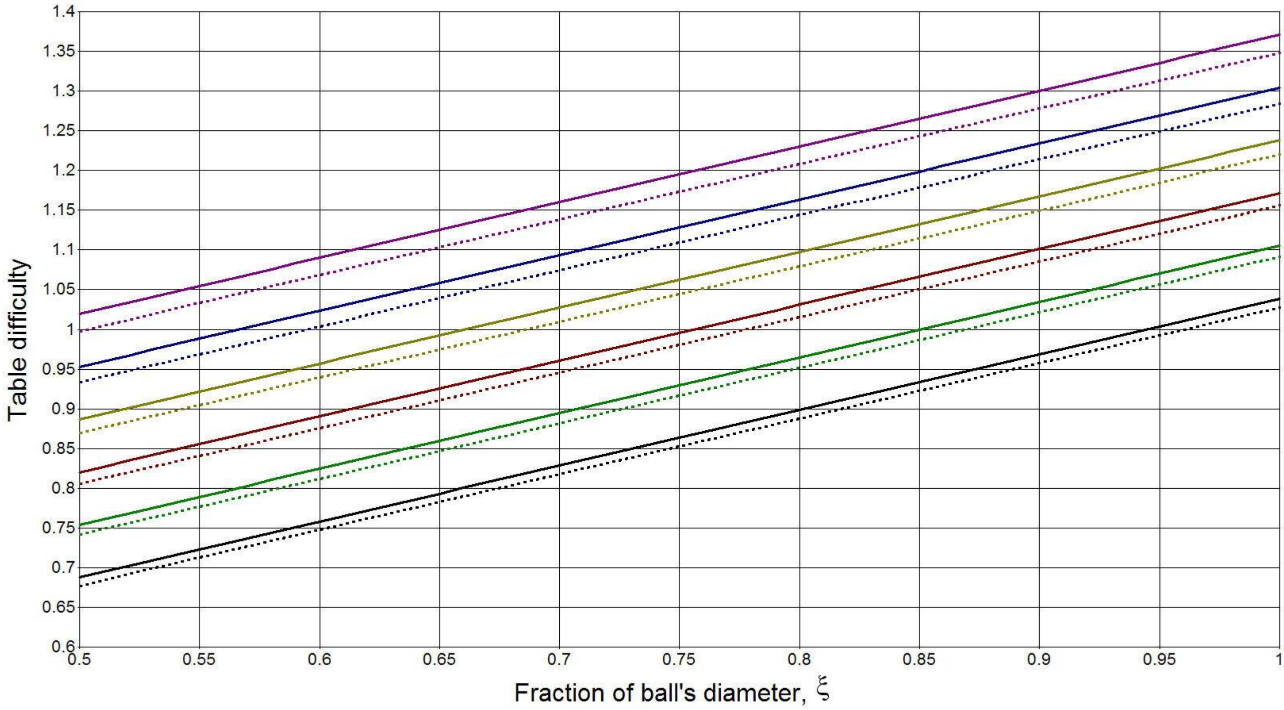

Fig.7. Difficulties for 12-foot and 10-foot billiard tables calculated using approach 2.

In my opinion, while determining the local pocket difficulty the preference should be given to the second of the considered approaches, and not to the first. The essence of the second approach, based on the use of virtual pocket width, is much easier to feel with «fingers»: you should just imagine how wide should be visible pocket opening that the ball could get into it from a particular starting point. Or, in other words, how far a magician would have to push the far point from the near knuckle.

Approach 3

The method proposed by V.Arkhipov (see above) is called the third approach for calculating estimates of table difficulty. In order the results obtained using approaches 1, 2 and 3 were consistent with each other we'll modify Arkhipov's formula and calculate the local pocket difficulty S_{ij} with respect to every initial location of the ball with coordinates (X_{O_i}, Y_{O_i}), $i=\overline{1,n}$, $j=\overline{1,m}$ according to the relation:

$$S_{ij} = 1 - \left[Arcsin\left(\frac{H_{ij} - 2R}{L_{ij}}\right) - Arcsin\left(\frac{H_{ij}}{L_{ij}}\right)\right],$$

where H_{ij} – visible pocket width; L_{ij} – distance from the ball to the point of pocket opening, which divides the viewing angle of the opening in half (principally under L_{ij} might be understood and the distance from the ball to the middle of pocket opening; S_{ij} value would change with slightly). Note, the relation can't be used for a number of points of the playing surface for which the following inequality $\left|\frac{H_{ij} - 2R}{L_{ij}}\right| > 1$ or $\left|\frac{H_{ij}}{L_{ij}}\right| > 1$.

The following Table 1 shows the results of difficulty 12-foot billiard table in comparison with similar estimates obtained by using approaches 1 and 2 (as a third approach does not imply the possibility of close contact of ball with far pocket point, then approaches 1 and 2 show only the results of calculations for $\xi = 1$). Note that the estimates obtained by using approaches 1 and 3 are quite close to each other. This is not surprising, since both approaches are based on the same value – the angular size of the visible width of pocket opening. It is very interesting (and rather surprising) that the difficulty of side pockets in all cases higher (and significantly) than difficulty of corner pocket.

Approach	Side pocket difficulty	Corner pocket difficulty	Table difficulty
1	1,792	1,056	1,301
2	2,001	1,056	1,371
3	1,774	1,055	1,295

Table 1. Local and integral difficulties for 12-foot Russian billiard table, calculated using three different approaches.

Approach 4

The pocket difficulty and can be evaluated by the ratio of the area of S_w of playing surface from which the ball can be potted into it (without touching the pocket knuckles or after the «allowable density» contact with far point) to the total area S_Σ:

$$S_k = 1 - \frac{S_W}{S_\Sigma}.$$

For calculation of the entire table difficulty, as in previous approaches, it is necessary to average amount of all pocket difficulties. I note that V.Arhipov attributed this approach to assessing table intake capacity.

The results of calculations performed by using the approach 4 are shown in Figure 8.

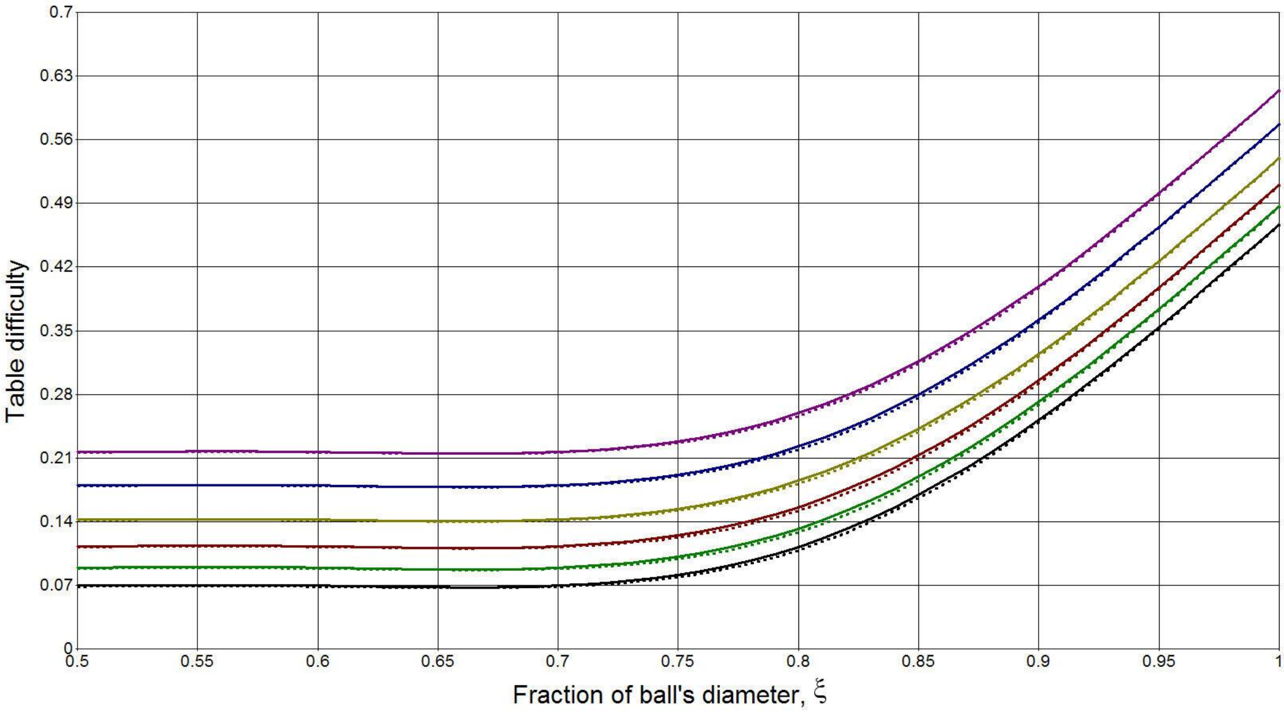

Fig.8. Difficulties for 12-foot and 10-foot billiard tables calculated using approach 4.

Comparison of tables for Russian Billiards, Snooker and Pool

Numerical results of table difficulties comparison for Russian Billiards, Snooker and Pool are presented in Table 2. Calculations were performed using approach 2. Because the Snooker and Pool tables have much more rounded knuckles than in Russian Billiards, for formalization the tasks of comparison some form of distorted pocket openings have been used. In fact, you could spend the time and «paint» a more accurate mathematical model for calculating the pocket difficulty with rounded points. But it seems that the effect of applying a refined model is not sensitive.

In the upper rows of the table you can see the parameters of tables and balls, corresponding to the standards of international billiard associations. The middle section shows the relative difficulties of corner and side pockets, representing a ratio of the diameters of balls used to pocket openings: $D\ /H$. These parameters correspond to the simplest representation of the pocket difficulties: pocket openings for Russian billiard tables comparable with the diameter of the ball; Snooker table pocket width is about one and a half times greater than the diameter of the ball, and Pool table – twice. Relative pocket difficulties correspond to the relative table difficulties $S_{rel} = \frac{1}{6}\left(\frac{4D}{Hcor} + \frac{2D}{Hside}\right)$. Ratios of relative table difficulty for Russian Billiards $S_{rel\ RB}$ to S_{rel} shown in the following line. It is easy to note that according to the values $S_{rel\ RB}\ /S_{rel}$ relative difficulty of Russian billiard table for about one and a half times greater than Snooker table difficulty and almost twice as high table difficulty for Pool. Pocket difficulties S_{cor} , S_{side} and table difficulties S, defined according to the second approach when $\xi = 1$ (for corner and side pockets), are presented in three lines below. In the bottom two rows of the table placed ratio values $S\,/S_{RB}$ and S_{RB}/S, where S_{RB} – table difficulty for Russian Billiards. It is easy to see that the $S_{RB}\ /S\ \approx S_{rel\ RB}\ /S_{OTH}$ as for Snooker table and for Pool. This means that using the calculation of integral table difficulty nothing new in tables' comparison have been received. As originally imagined (using sizes of balls and pockets), and it has turned: Russian billiard table for about one and a half times stricter Snooker table and twice stricter than Pool table. (Incidentally, in the above-mentioned article Dr. Dave gave a link to a billiard forum *AZBilliards*, where he posted a greater amount of information concerning the comparison of different table difficulties. If you closely examine cited therein numerical data, it is possible to extract assessment difficulty for Russian 12-foot table relative to a standard Pool table. This estimate is equal to 1.84, is quite close to the value of 1.907 as shown in table. It should be emphasized that in order to find its assessment of Dr. Dave applied a completely different approach).

	12-foot table for Russian Billiards, complies with standards of ICP and ECP	12-foot table for Snooker, complies with standards of WPBSA	9-foot table for Pool, complies with standards of WPA
Table length, mm	3550	3569	2540
Table width, mm	1775	1778	1270
H_{cor} – corner pocket mouth size, mm	72	82.55	117.5
H_{side} – side pocket mouth size, mm	82	99.212	130.175
D – diameter of the ball, mm	68	52.5	57.15
Cushion nose height, mm	42	26.25	36.29
D / H_{cor} – relative difficulty for corner pocket	0.944	0.636	0.5
D / H_{side} – relative difficulty for side pocket	0.829	0.529	0.45
S_{rel} – relative table difficulty	0.906	0.6	0.483
$S_{rel\,RB} / S_{rel}$	1	1.51	1.876
S_{cor} – corner pocket difficulty	1.056	0.713	0.563
S_{side} – side pocket difficulty	2.001	1.331	1.031
S – table difficulty	1.371	0.919	0.719
S / S_{RB}	1	0.67	0.524
S_{RB} / S	1	1.492	1.907

Table 2. Comparison of characteristics of tables for Russian Billiards, Snooker and Pool.

Comparison of the table difficulties for Russian Billiards, calculated by varying the width of pocket openings and diameter of the ball

Because the comparison of the table difficulties for various kinds of pocket billiard games almost did not bring anything new to its understanding, there was consideration of comparing the table difficulties, the corresponding allowable variation in pocket sizes and ball diameters. For 12-foot table on which Russian Billiards are played, small but sensitive spreads of pocket and ball parameters is allowed. Corner pocket can be made in widths from 72 to 73 millimeters, and the width of the side pocket should be 82 ÷ 83 mm. Nominal diameter of a billiard ball is 68 millimeters, but it is permitted to use the balls with a diameter of up to 68.5 millimeters. Table 3 shows the results of calculations performed in «extreme» situations: for the most narrow pockets in conjunction with the largest diameter ball; for the widest pockets in conjunction with the smallest ball. It can be seen that the calculated table difficulties differ from each other by about 2 percent. On the one hand, there is a slight difference between the values of the integral table difficulty, and on the other hand – a cardinal difference in gaming properties: one of the considered tables is very difficult to play and the other can be called «gratuitous». This is further evidence of the ineffectiveness of using the table difficulty parameter in order to characterize its real gaming difficulty.

Corner pocket mouth size, mm	Side pocket mouth size, mm	Diameter of the ball, mm	Corner pocket difficulty	Side pocket difficulty	Table difficulty
72	82	68.5	1.063	2.016	1.381
73	83	68	1.041	1.977	1.353

Table 3. Characteristics of 12-foot table for Russian Billiards.

Local pocket difficulties of 12-foot table for Russian Billiards

In the above approaches, local pocket difficulties were determined only with the purpose to use them for finding an estimate of the integral table difficulty. But, as it turned out, this assessment was not «weighty» to make judgments about the characteristics of the table and to compare different tables – in any case, in a superficial analysis. Therefore, it seems logical to take a closer «look» to the estimates of local pocket difficulties.

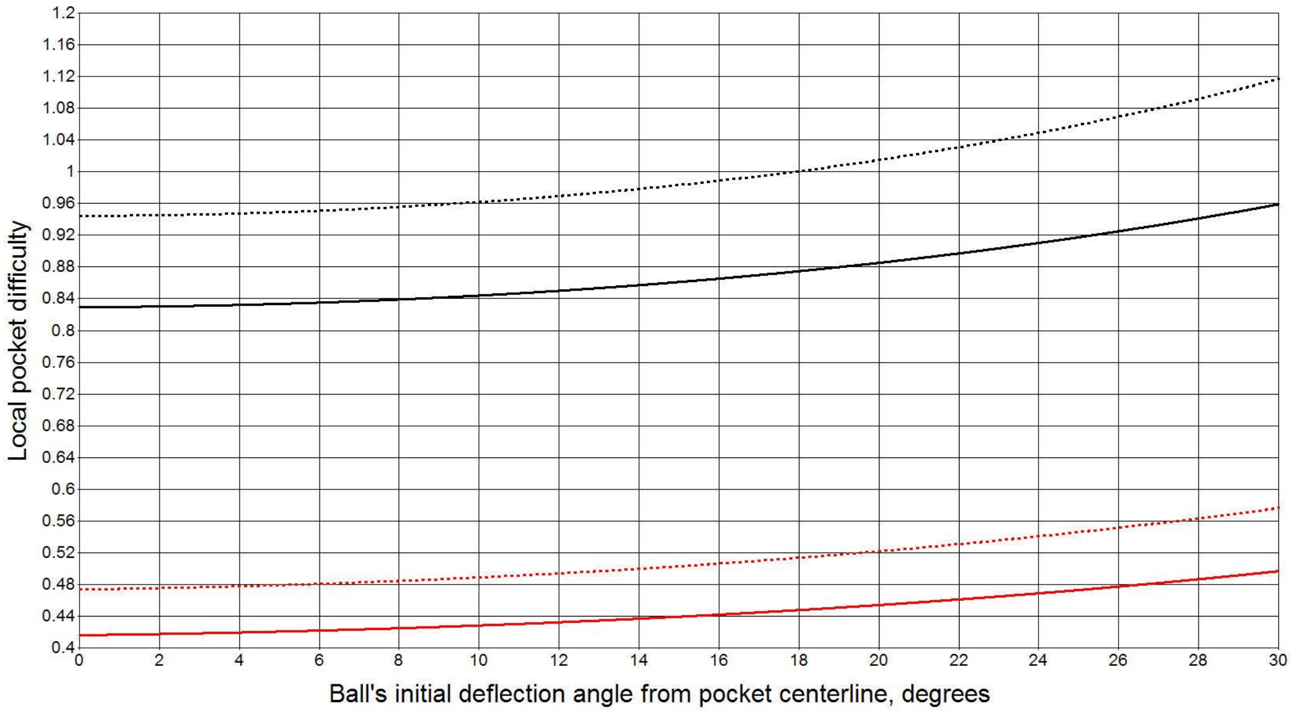

Fig.9. Local pocket difficulties of 12-foot table as a function of the ball's initial deflection angle from pocket centerline.

Let us have a look at the results of the calculations presented in Figure 9. The horizontal axis (x-axis) represents values of ball's initial deflection angle α from pocket centerline. Under the α is understood an angle between centerline and the line drawn from the point of initial ball position to the heart (dead center) of the pocket. It is clear that for the vast majority of positions of the ball on the table $\alpha \neq 0°$. Therefore it is interesting – how the local pocket difficulty changes as a function of the ball's angular distance towards the pocket centerline. Values of pocket difficulties are plotted on the y-axis. Graphs shown in solid lines correspond to the side pocket, and dashed lines – the corner pocket. Values of local difficulties provided in the graphs red designed for corner and side pockets at $\xi = 0.5$. Black is the color of graphs corresponds to the calculations, which were used value $\xi = 1$.

Choosing shot players willingly inclined to the positions of the ball, in which $\alpha \approx 0°$. Decision to attack the pocket usually accepted if $\alpha \lesssim 15°$, but sometimes not «disdain» and positions $15° < \alpha \lesssim 30°$. Attacking shots are extremely rare when $\alpha > 30°$. Figure 9 shows that in the considered range of $0° \leq \alpha \leq 35°$ local corner pocket difficulties about 15% higher than local difficulties of side pocket. And this is not surprising, because the side pocket still wider than corner pocket. Rather, it might seem somewhat surprising that the side pocket difficulty with respect to the entire playing surface significantly exceeds the corner pocket difficulty (which was mentioned above). But if you get a good think about it, it is within the usual framework of understanding – in fact into the corner pocket, you can sent a ball located virtually anywhere on the table, but in relation to side pocket it is not true. From figure 9, it also follows that a decrease in the value ξ from 1 to 0.5 (this means that instead of touching far point in passing allowed close contact up to half-ball collision) local pocket difficulty falls by about half.

To judge how much pocket becomes stricter locally as compared with the case $\alpha = 0°$, you can see in Figure 10. In other words, assigning a value of $\alpha \neq 0°$, we can estimate – how much harder to pot the ball into the pocket in relation to the case when it was originally located on the pocket centerline. For example, when $\alpha = 35°$, $\xi = 0.5$ to send the ball into the corner pocket by almost a third (30%) is more difficult than it would be in the case when the initial position of the ball coincides with pocket centerline.

It is easily seen that the functions of the relative difficulty illustrated in Figure 10, have a monotonic increasing positive derivative (in other words, their growth rate increases with increasing α). This means for example that when $\alpha = 10°$ pocket stricter relative to the case $\alpha = 0°$, than when $\alpha = 20°$ in comparison with the case of $\alpha = 10°$. This trend, in particular, leads to the fact that players often prefer to opt for an attack shot significantly distant from the pocket the object ball angled $\alpha = 10°$, and not close to a pocket ball with $\alpha = 20°$.

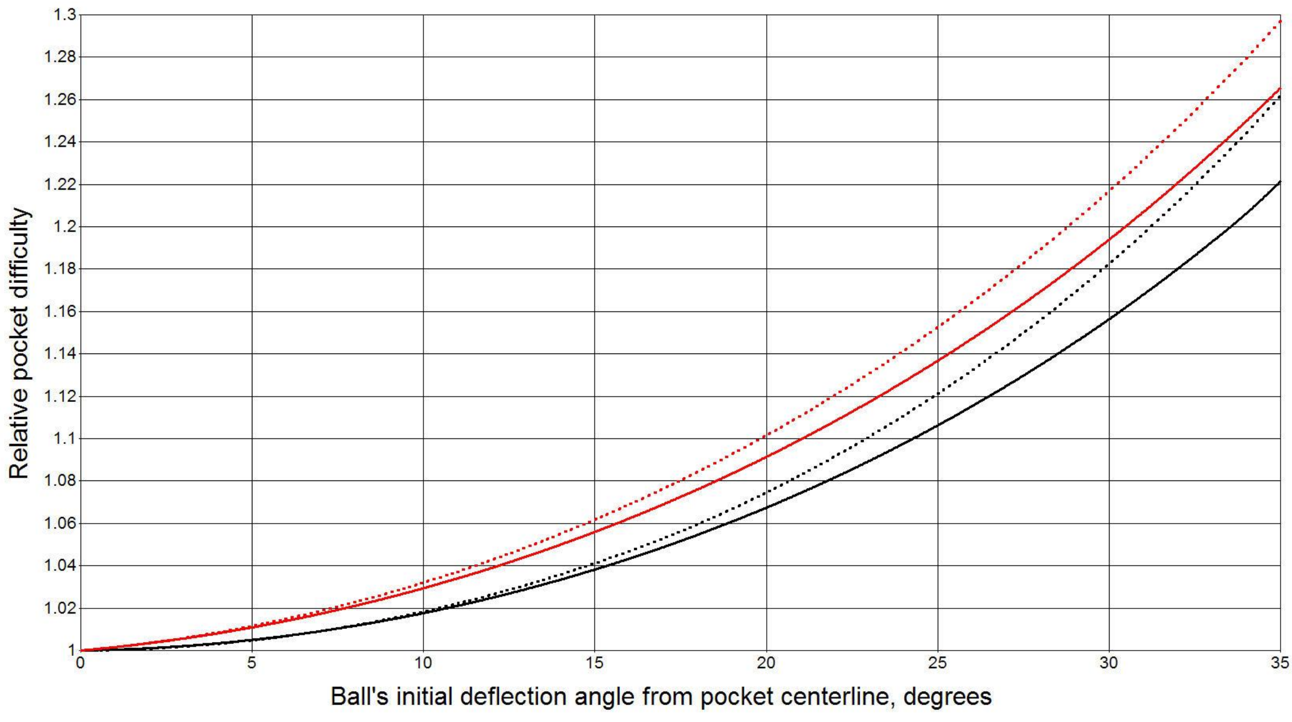

Fig.10. Local pocket difficulties for 12-foot table relative to the case $\alpha = 0°$.

Comparison of local pocket difficulties for Snooker and Pool tables with pockets for Russian billiard table

How pockets for Russian Billiards table stricter than pockets for Snooker and Pool tables? Partially you can answer this question by using the numerical values of the relative pocket difficulty, shown in Table 2. It's easy to calculate that corner pocket table for Russian Billiards stricter than Snooker corner pockets to 1,484 times and stricter than corner pocket for Pool to 1,888 times. For side pockets these same ratios are 1.567 and 1.842, respectively. However, these values characterize the relative local pocket difficulty only for the case $\alpha = 0°$. And what will happen when $\alpha \neq 0°$? The answer can be found by using numerical results presented below in Figures 11 and 12. Perhaps we can say that in the angular range $0° \leq \alpha \leq 30°$ ratios of local pocket difficulties vary slightly. You can notice a strange, at first glance, trend: with increasing angle α, the relative pocket difficulty for Russian billiard table compared with the pocket difficulty for Snooker and Pool decreases.

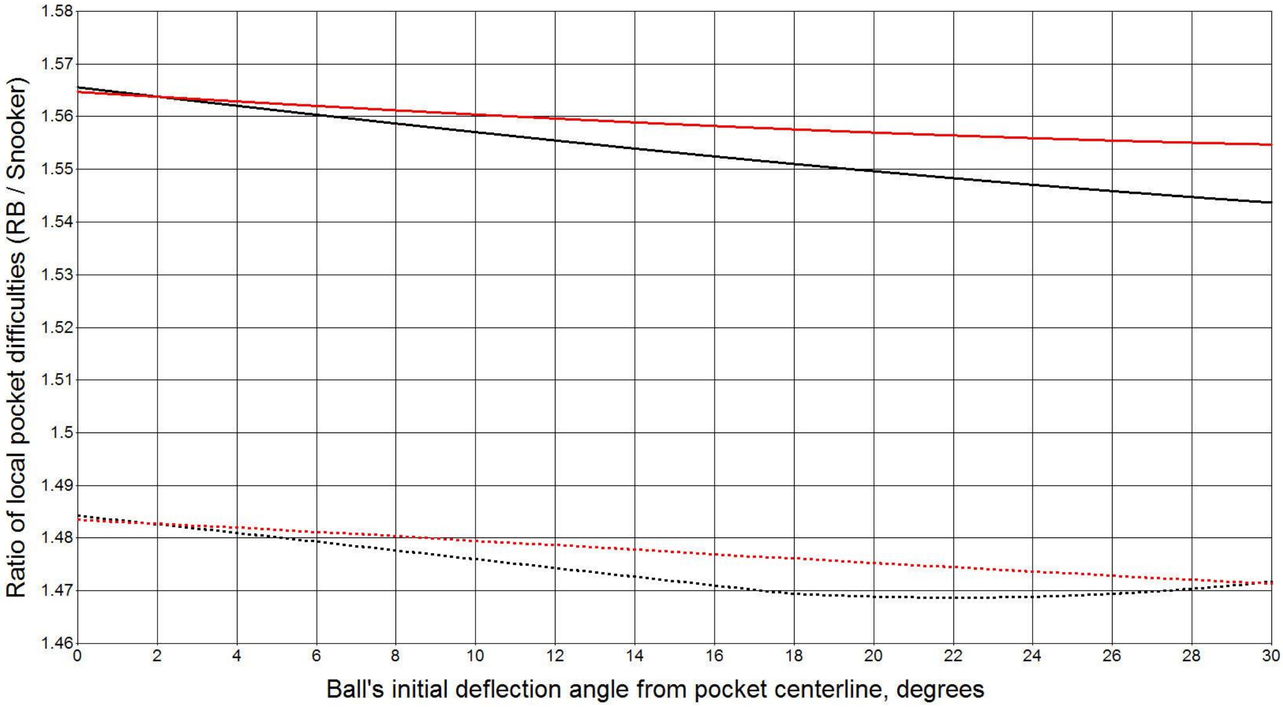

Fig.11. Ratios of local pocket difficulties for Russian Billiards to local pocket difficulties for Snooker.

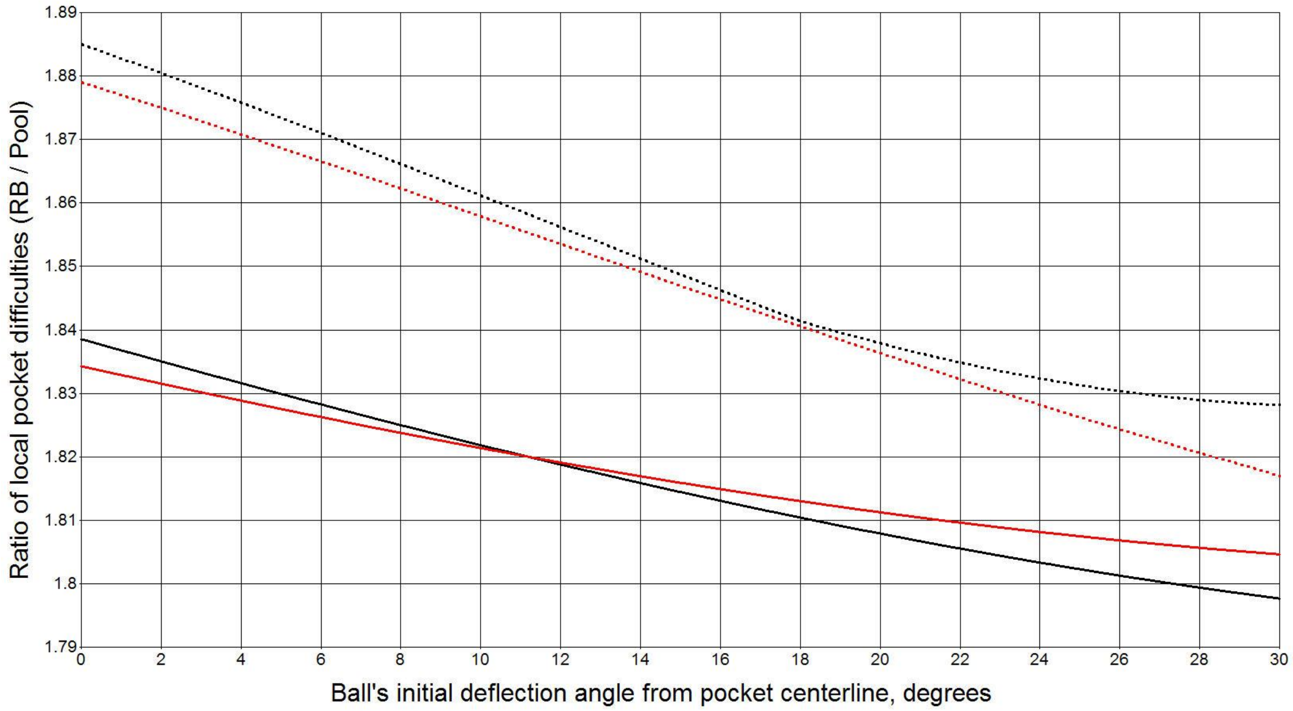

Fig.12. Ratios of local pocket difficulties for Russian Billiards to local pocket difficulties for Pool.

Conclusions

1. 10-foot table difficulty for Russian Billiards is only about two percent less than standard 12-foot table difficulty.
2. Integral side pocket difficulty (calculated by reference to the playing surface for Russian Billiards) is much higher than the corner pocket difficulty (more than half).
3. Table for Russian Billiards about half stricter than Snooker table and almost twice stricter than Pool table. Such estimates can be obtained without using the concept of integral difficulty, but simply comparing the sizes of pocket openings and sizes of balls. In this sense, the evaluation of the integral table difficulty doesn't give additional information.
4. For most positions in the game of Russian Billiards local corner pocket difficulty about 15 percent exceeds local side pocket difficulty.
5. Purely geometric approach does not allow a meaningful estimate of the true playing difficulty for billiard tables.

Part 2

May, 2014

Radius of the ball defines two important billiard game parameters: the size (dimensions) and the curvature of the surface. In the first part of this work when calculating the billiard table difficulty only the dimensions of the ball were taken into account. It is through them the possibility of penetration of the ball through the visible pocket opening is determined. The second parameter (curvature) affects the trajectory of balls after the collision. It is clear, that while playing billiards, people invariably make mistakes when striking shots. In particular, no matter how carefully the player would take aim and how accurately he would try to make shot the cue ball still hits the object ball not at the point, which was planned in theory. As to good players collision error is usually small but beginners can reach significant values. Nevertheless, we can safely say that such errors will always be present.

Collision error entails error in the direction of movement of the object ball (and, naturally, the cue ball). I think many people pay attention to the fact that while playing smaller balls players have to perform more accurate shots – or the real trajectory of balls will be very far from the expected. This is a consequence of the fact that smaller balls have a greater curvature of the surface. Let us consider the patterns of influence of curvature on the accuracy of the shots.

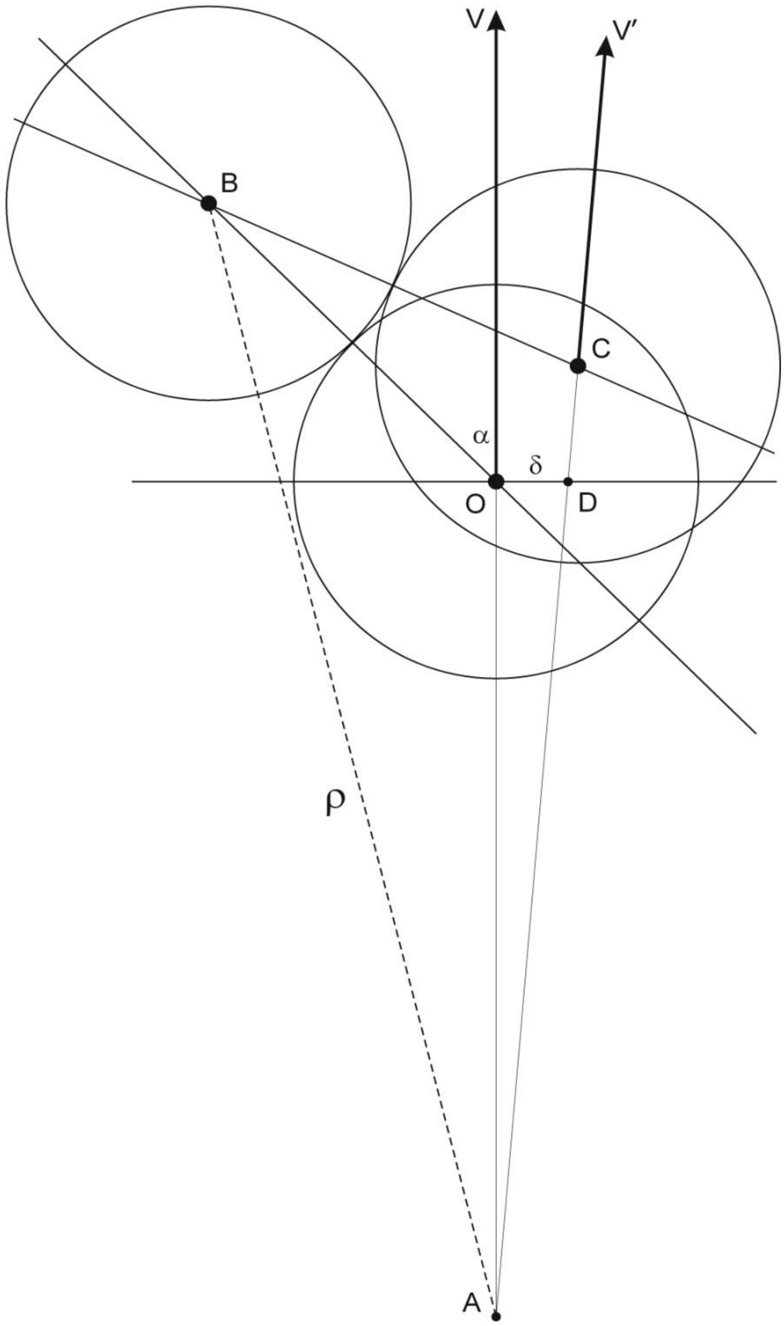

Fig.13. Collision of balls – the nominal case and the case of overcut.

Look at to Figure 13, which schematically depicts two situations of collision. The initial position of the cue ball, determined by the point A, is the same for both situations. <u>The first collision situation</u> in which the center of the cue ball takes the position O, is nominal. In other words, it is assumed that the player would like to send the cue ball just in this position. The ball-hit fraction is determined by the cut angle α, which form the translational velocity vector V of the cue ball and the line of centers, passing through the centers of the balls O and B. It is clear that $\alpha = 0°$ for straight-in shot and $\alpha = 90°$ for the collision in passing. In a typical case of a half-ball collision, when the vector V passes through the edge of the object ball, cut angle is thirty degrees. The second situation is different from the first impact because the player allowed a lateral miss equal to δ and reflected by segment OD. Since in this case the actual cut angle exceeds α, then the second situation corresponds to the case of overcut.

Due to that error δ, the real line of centers, passing through points C and B, will be inclined to the nominal line of centers OB at angle $\gamma = < OBC$. This angle depends on the curvature of the balls will generate a miss of the object ball when moving to the target (usually to the pocket). Express the angle γ of the ball radius R, cut angle α, miss of the cue ball δ and the initial distance between the balls ρ, equal to the length of the segment AB in Figure 13. From the triangle ABO by the law of sines follows the relation $\frac{AB}{Sin(180° - \alpha)} = \frac{2R}{Sin < BAO}$, from which we can find the angle BAO:

$$< BAO = Arcsin\left(\frac{2R\ Sin(180° - \alpha)}{\rho}\right). \qquad (16)$$

Accordingly, we can determine the angle ABO: $< ABO = \alpha - < BAO$. Applying to the same triangle ABO the law of cosines, we'll find the dependence $AO^2 = \rho^2 + (2R)^2 - 2 \cdot \rho \cdot 2R \cdot Cos < ABO$, from which it is easy to determine the length of the segment AO. Knowing the value of AO you can find the angles OAD and BAC: $Tg < OAD = \delta / AO$; $< BAC = < BAO + < OAD$. From the triangle BAC to be the law of sines $\frac{AB}{Sin < ACB} = \frac{2R}{Sin < BAC}$, where you can find the angle ACB:

$$< ACB = Arcsin\left(\frac{\rho\ Sin < BAC}{2R}\right); < ACB \in (\pi/2, \pi). \qquad (17)$$

Now it is easy to determine $< ABC = 180° - < BAC - < ACB$ and desired angle $\gamma = < OBC = < ABC - < ABO$.

Let us now consider Figure 14, which shows the nominal collision situation and the situation corresponding to the undercut case. Thus the real cut angle is less than the nominal value α. Let us express the angle γ. Similarly, as was done above, we define the angles BAO and ABO, the value of the AO and the angle OAD. Find an angle BAC: $< BAC = < BAO - < OAD$. The angle ACB is determined according to (17) and then computes the angle ABC: $< ABC = 180° - < BAC - < ACB$. Finally, the desired angle γ is as follows: $\gamma = < OBC = < ABO - < ABC$. Knowing the angle γ, we can easily calculate miss of the object ball (its deviation from the nominal line of centers) Δ at a distance S from the original location: $\Delta = S \cdot Tg\, \gamma$. Figure 15 shows the dependence of Δ on the cut angle α, calculated at $\rho = 1$ m; $S = 1$ m; $\delta = 1$ mm. Red lines correspond to Russian Billiards balls with a radius of 34 mm, and the black lines – balls for Snooker in which $R = 26.25$ mm. The curves shown by solid lines are obtained for the cases of overcut, and dashed lines for undercut cases. It is easy to see that with increasing angle α values of Δ increase. Especially, this tendency is clear for cut angles $\alpha > 45°$.

Figure 15 clearly shows that misses Δ for balls used in Snooker significantly exceed misses corresponding balls for Russian billiards. For example, the error δ which is equal to one millimeter, when hitting the half-ball of Russian Billiards leads to miss of the object ball at a distance of $S = 1$ m, component $\Delta = 17$ mm. And the same error in Snooker entails miss $\Delta = 22$ mm. For comparison of table difficulties so interesting relative misses equal to ratios of quantities Δ, corresponding to Russian Billiards Δ_{RB} and Snooker Δ_S: $K_S = \Delta_{RB} / \Delta_S$.

Dependence of the coefficient K_S on cut angle α presented in Figure 16. For the estimations can be assumed K_S constant. For example, we can assume that $K_S \approx 0.772$.

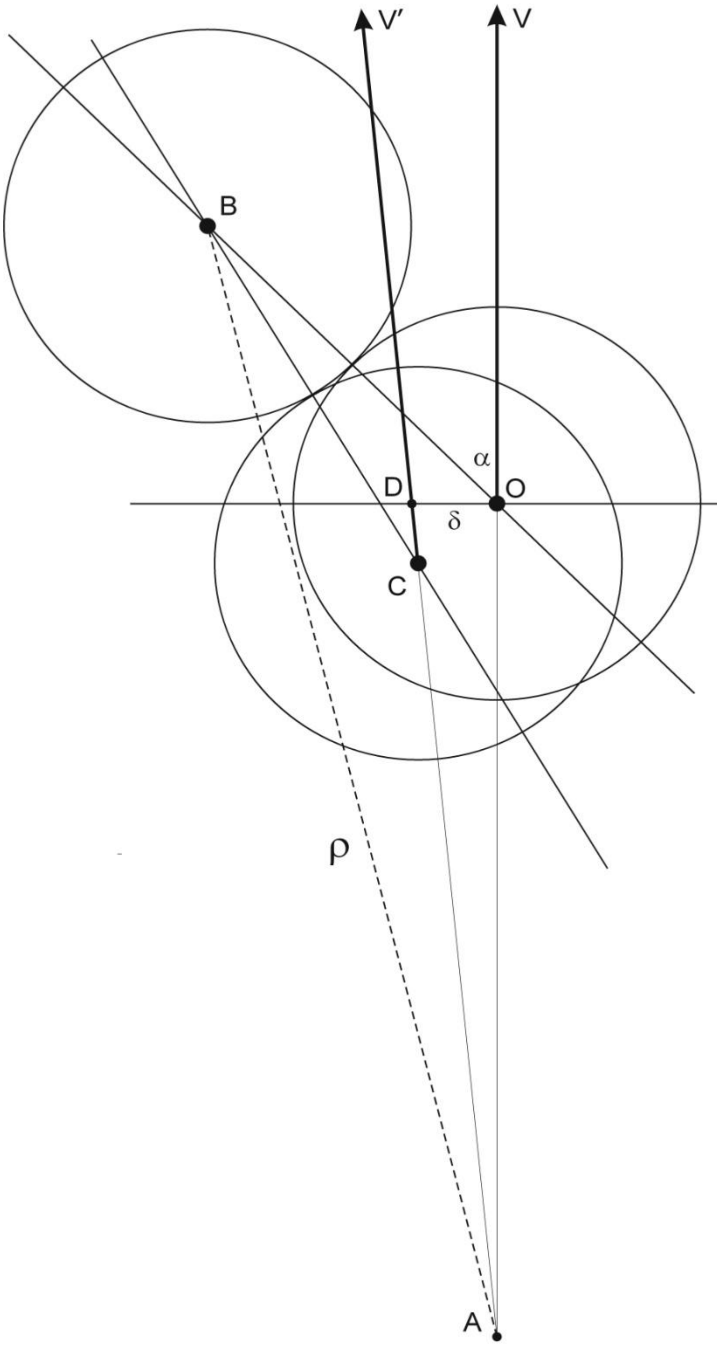

Fig.14. Collision of balls – the nominal case and the case of undercut.

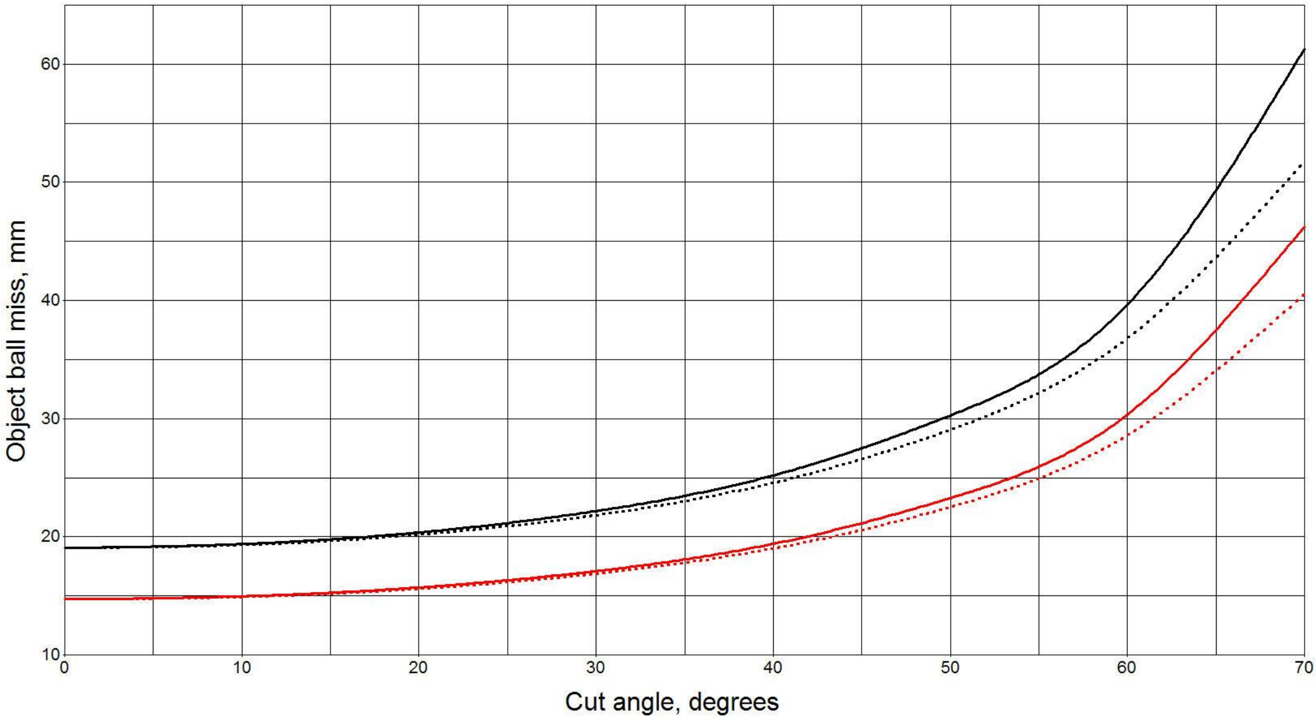

Fig.15. Russian Billiards and Snooker: miss Δ dependence on cut angle α.

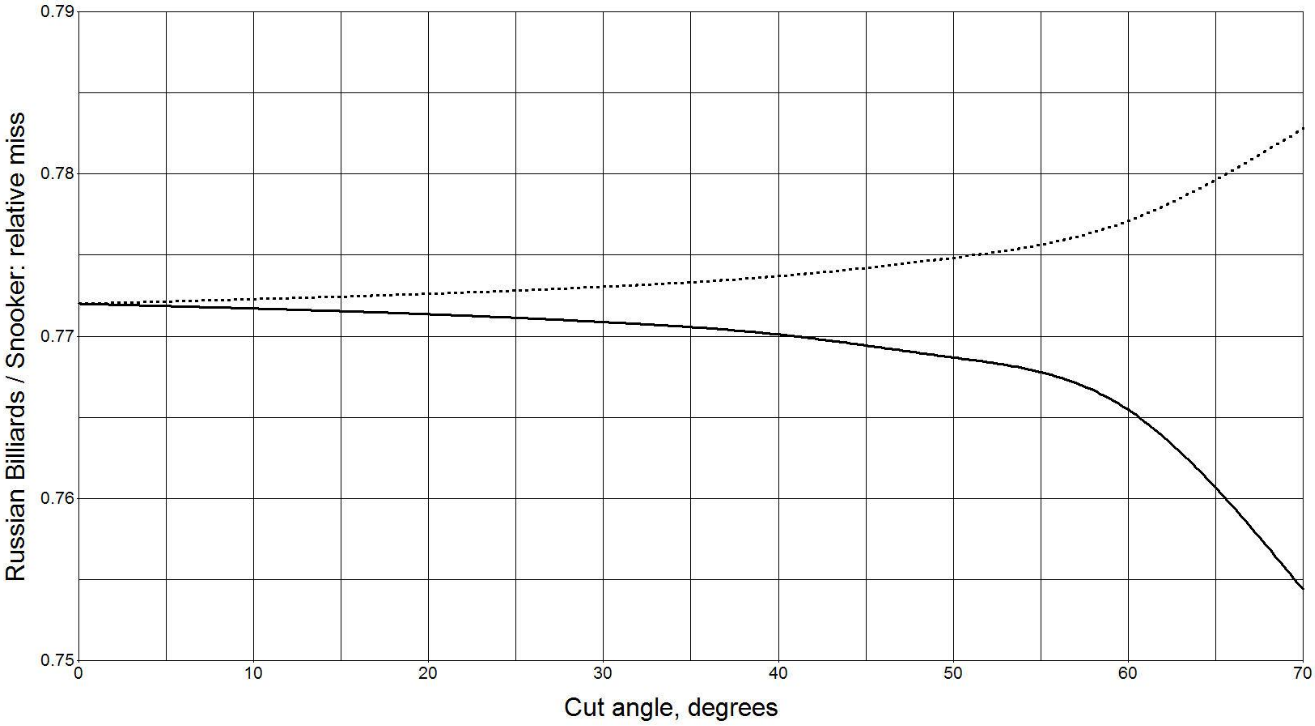

Fig.16. Russian Billiards and Snooker: relative miss dependence on cut angle α.

Figure 17 shows the dependence of Δ on the initial distance between the balls ρ. In the calculations it is assumed that a collision occurs at half-ball hit, that is, when $\alpha = 30°$. From the figure clearly shows that the influence of the magnitude of the value of ρ on Δ can be neglected.

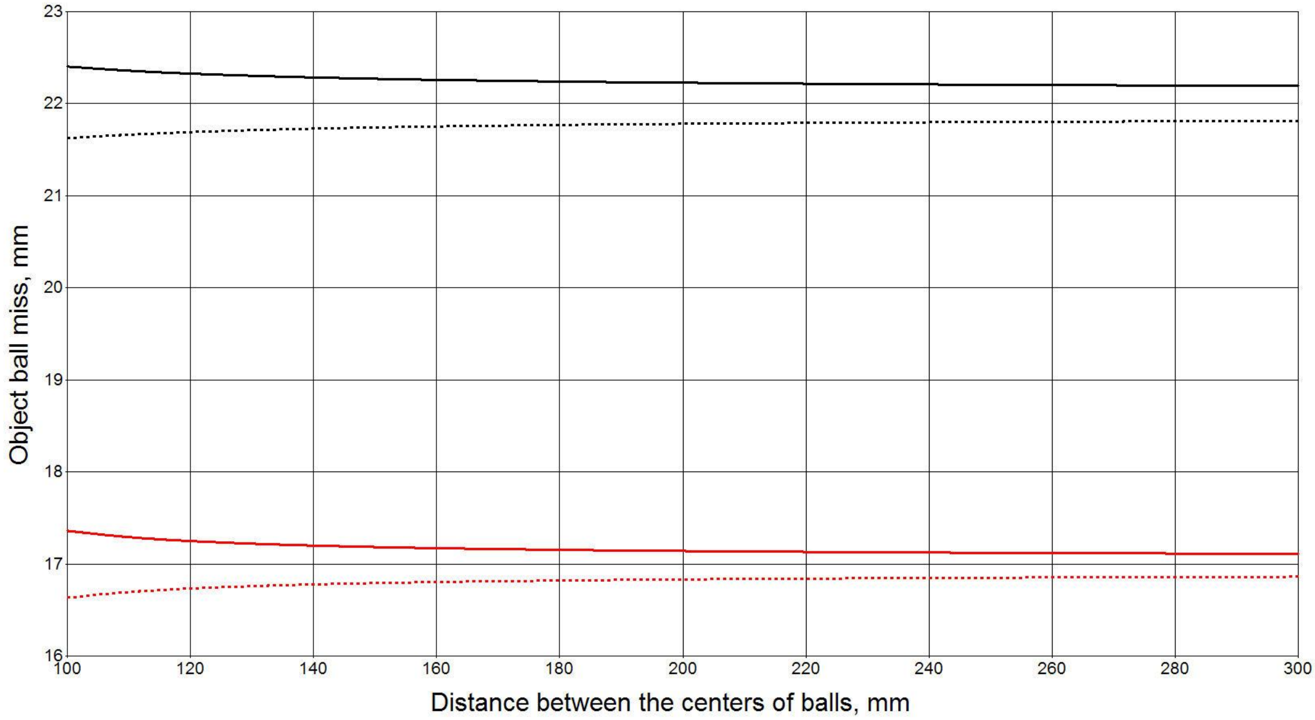

Fig.17. Miss of object ball dependence on initial distance between centers of balls.

Similar calculations were carried out for comparison tables for Russian Billiards and Pool. In the calculations it was assumed that the radius of the balls for a Pool game is 28.575 mm. Figure 18 shows the dependences of Δ on α, as in Figure 19 – dependences of $K_P = \Delta_{RB} / \Delta_P$ on α. It is easy to see that we can assume $K_P \approx 0.841$.

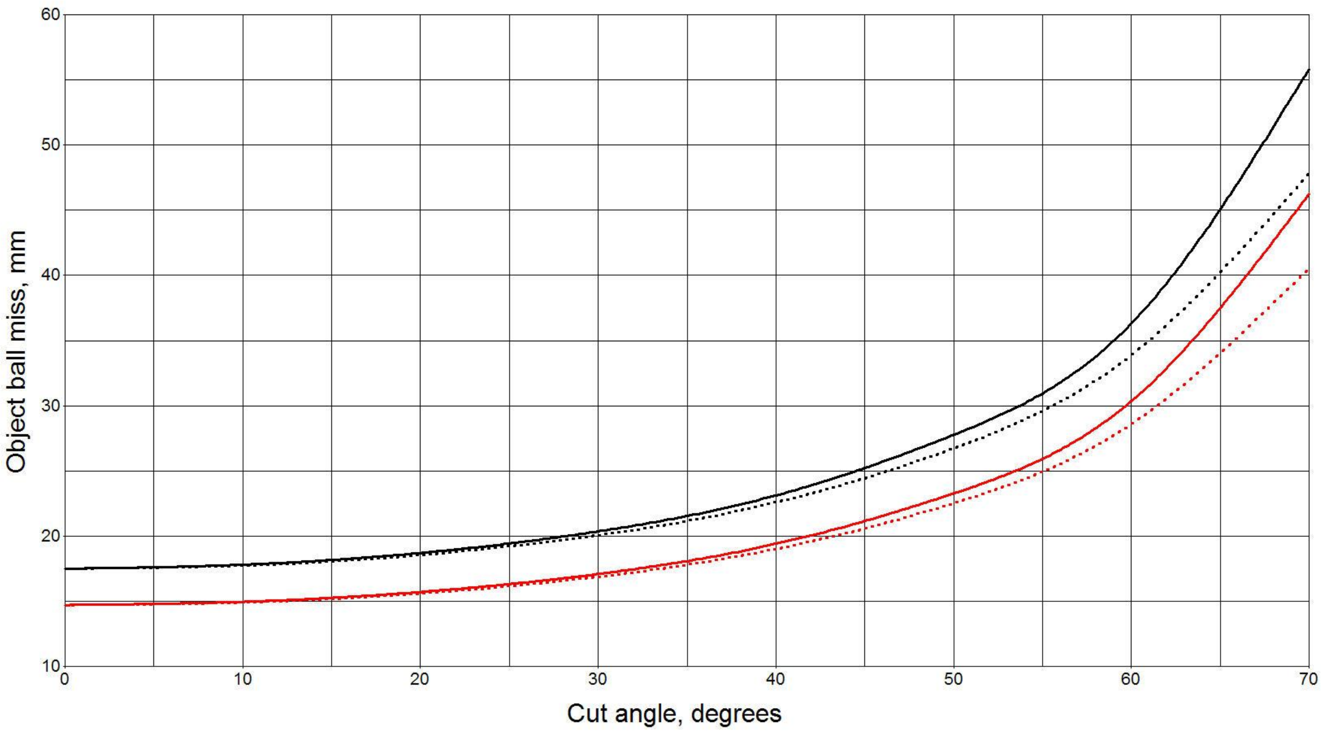

Fig.18. Russian Billiards and Pool: miss Δ dependence on cut angle α.

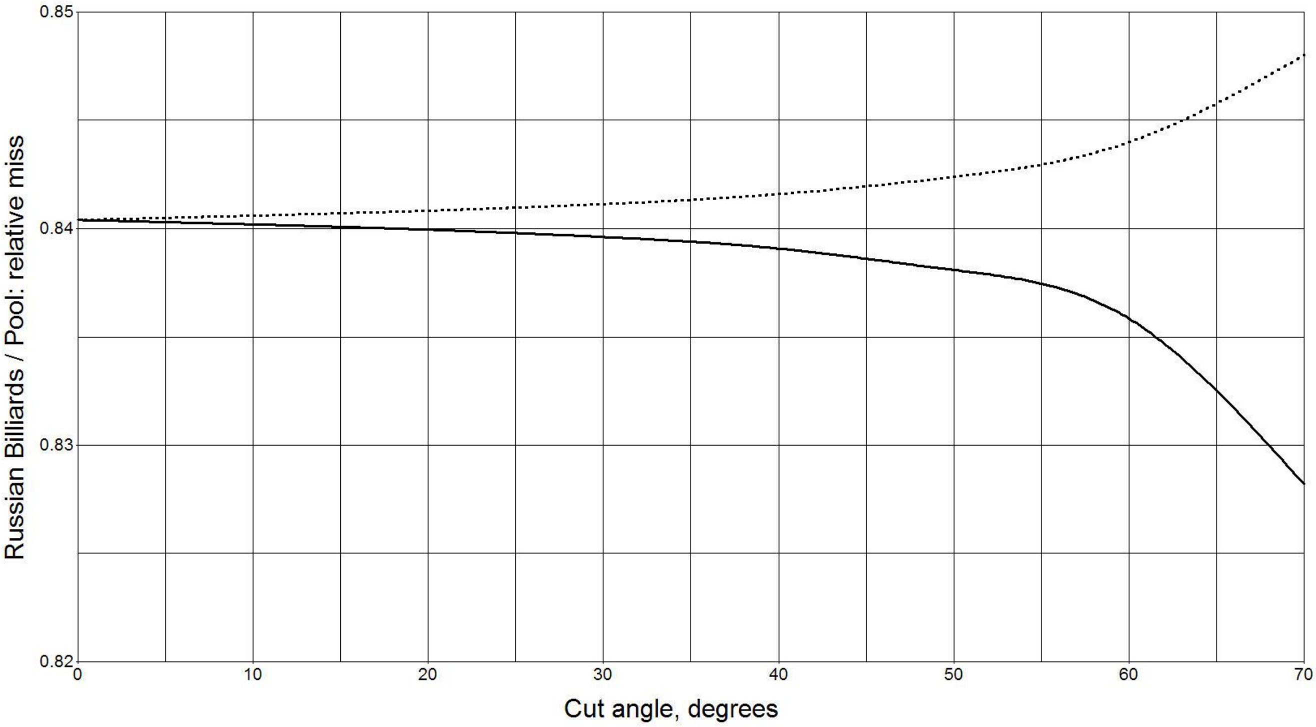

Fig.19. Russian Billiards and Pool: relative miss dependence on cut angle α.

In the first part of this article were found coefficients of billiard table relative difficulty (see Table 2). Thus, the table difficulty for Russian Billiards with respect to a Snooker table characterized with coefficient $C_{\mathrm{S}} \approx 1.492$. In relation to Pool table was found C_{P} factor ≈ 1.907. Using these coefficients and quantities of K_{S} and K_P we can calculate more realistic table difficulties: $C_S' = C_{\mathrm{S}} \cdot K_{\mathrm{S}} = 1.492 \cdot 0.772 \approx 1.15$; $C_P' = C_{\mathrm{P}} \cdot K_{\mathrm{P}} = 1.907 \cdot 0.841 \approx 1.6$.

Conclusion

Table for Russian Billiards is stricter than Snooker table only about 15 percent. Russian Billiards Table difficulty exceeds Pool table difficulty by about 60 percent. The resulting estimates give an idea of the relative difficulties of billiard tables that differ significantly from prevailing stereotypes.